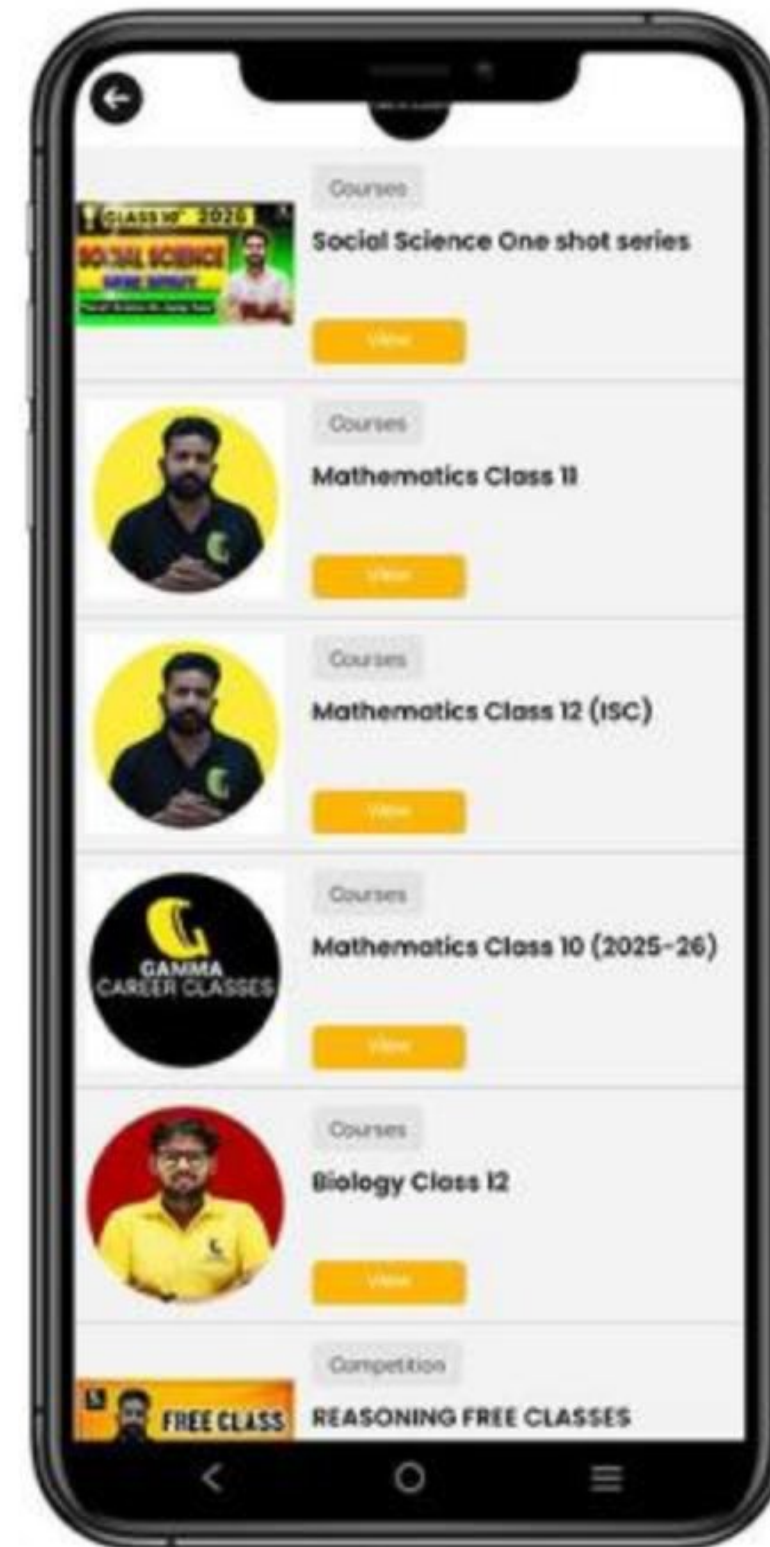
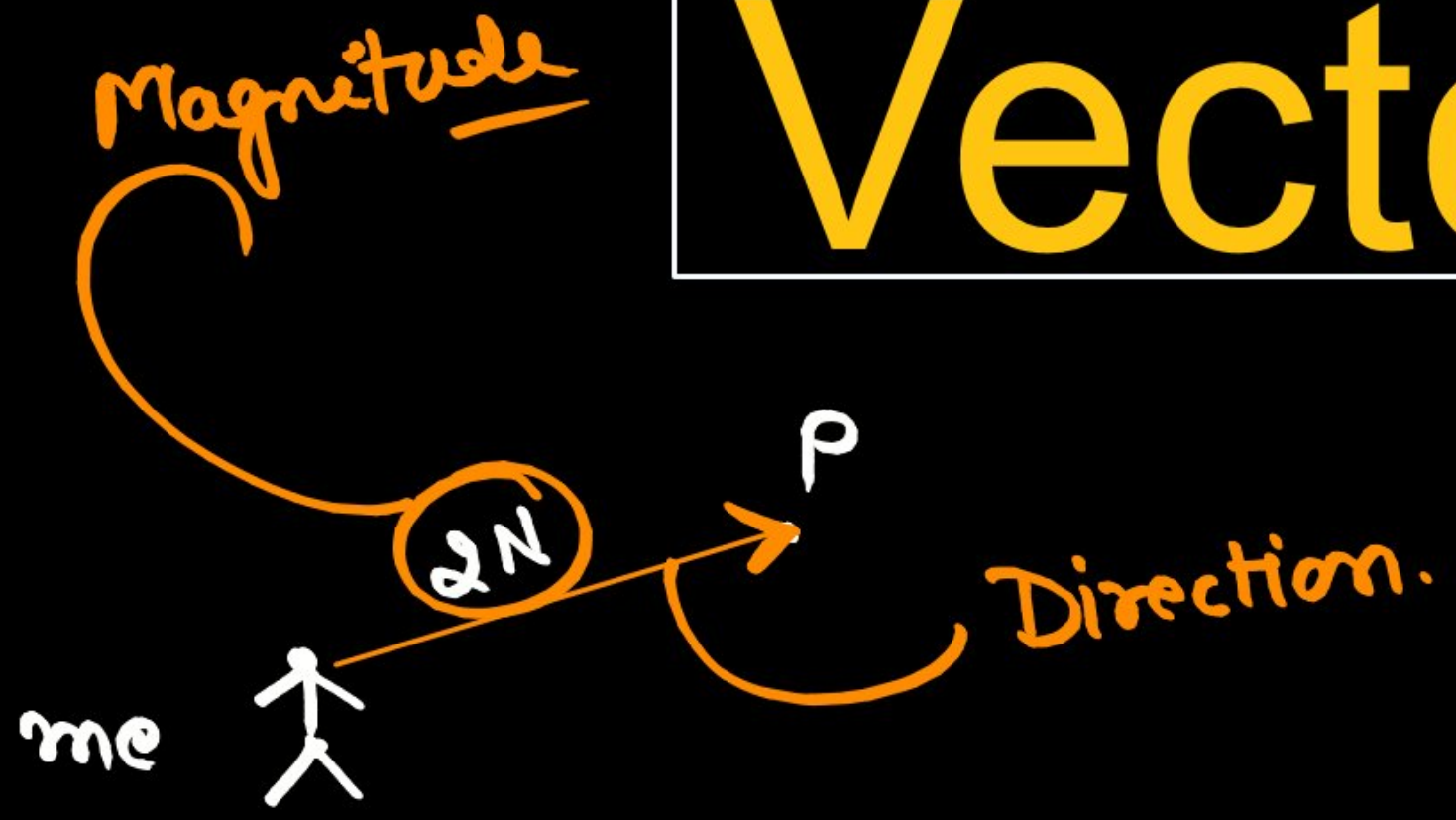


Vector

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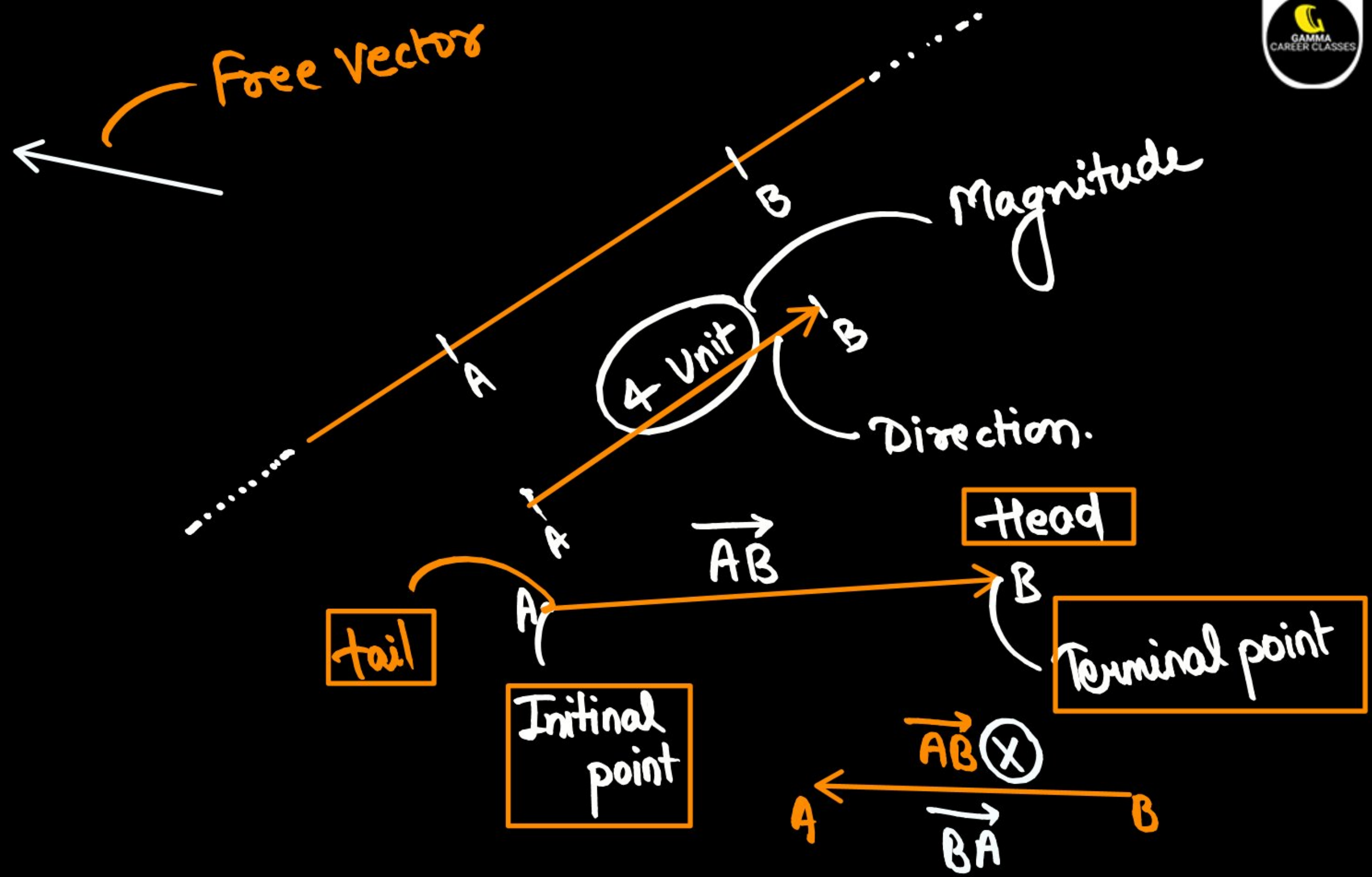
Vector



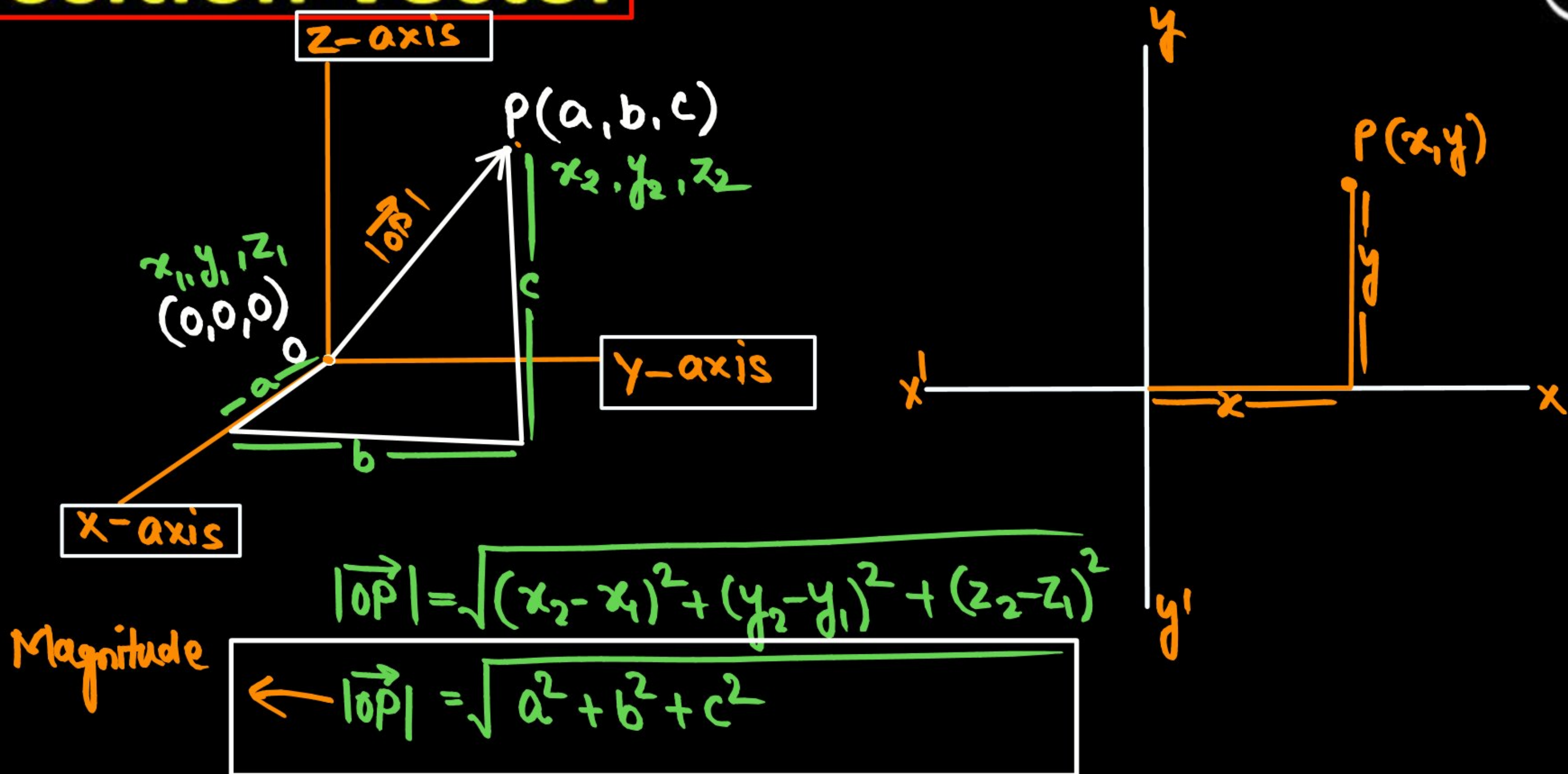
• Q

Magnitude + Direction bhi hogi

Vector Quantities



Position Vector



Direction Ratio's and Direction Cosine

$\alpha, \beta, \gamma \rightarrow$ Directed angle

$$\begin{matrix} \cos \alpha & \cos \beta & \cos \gamma \\ l & m & n \end{matrix} \quad \text{D.R.}$$

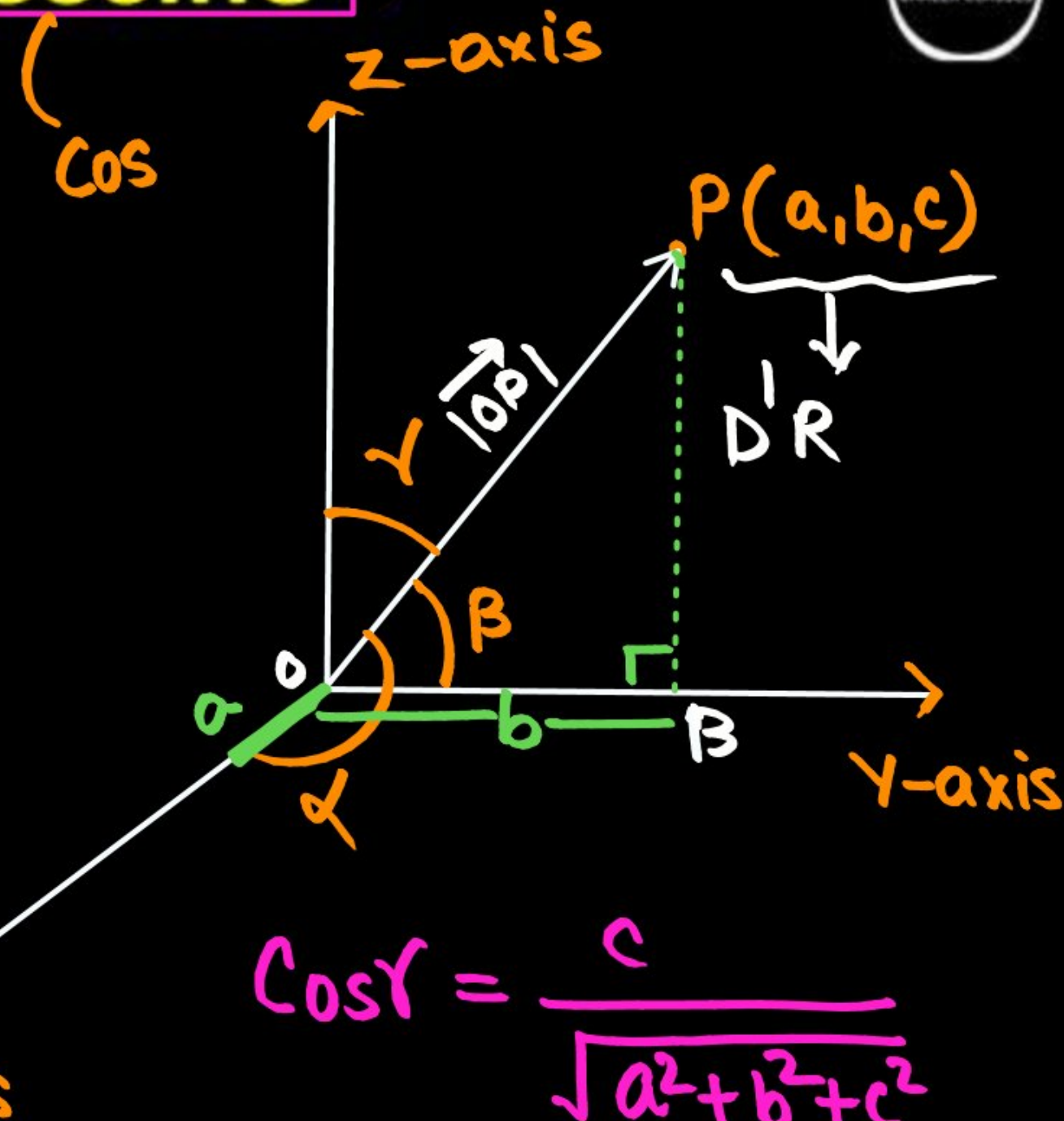
$$|\vec{OP}| = \sqrt{a^2 + b^2 + c^2}$$

In $\triangle OBP$

$$\cos \beta = \frac{OB}{OP} = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$\cos \alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$\cos \gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$



$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

OR

$$l^2 + m^2 + n^2 = 1$$



$$\left[\frac{a}{\sqrt{a^2 + b^2 + c^2}} \right]^2 + \left[\frac{b}{\sqrt{a^2 + b^2 + c^2}} \right]^2 + \left[\frac{c}{\sqrt{a^2 + b^2 + c^2}} \right]^2$$

$$\frac{a^2}{a^2 + b^2 + c^2} + \frac{b^2}{a^2 + b^2 + c^2} + \frac{c^2}{a^2 + b^2 + c^2}$$

$$\frac{a^2 + b^2 + c^2}{a^2 + b^2 + c^2} = 1$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

D'R and D'C proportional

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k$$

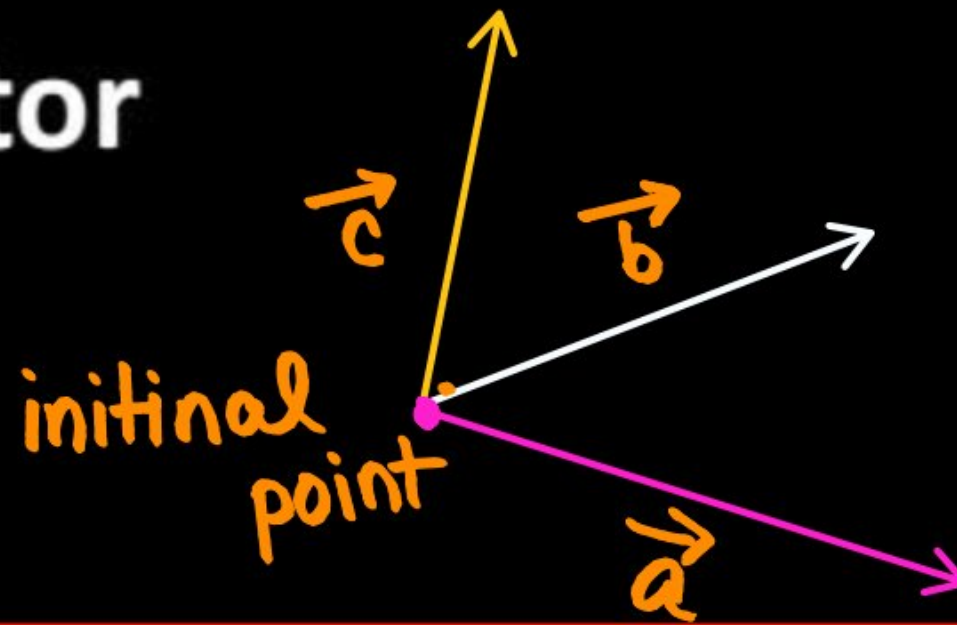
Types of Vector



Zero Vector $\rightarrow$ Magnitude = 0
 $\vec{AA}, \vec{BB}$

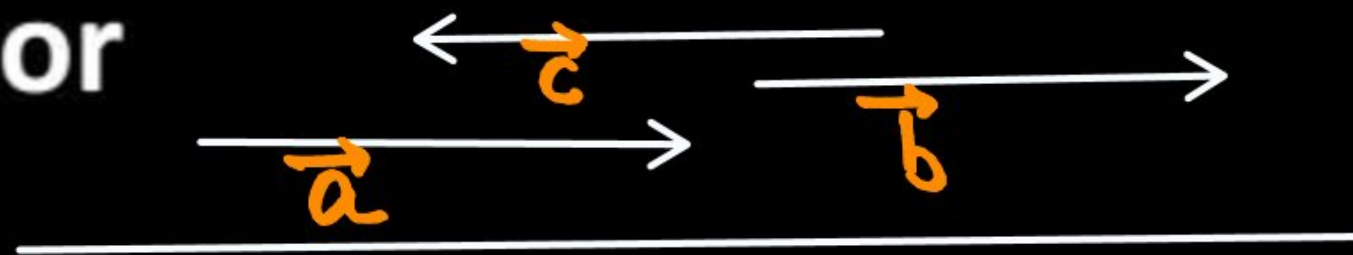


Coinitial Vector



$$\vec{a} = a_1\hat{i} + b_1\hat{j} + z_1\hat{k}$$
$$\vec{b} = a_2\hat{i} + b_2\hat{j} + z_2\hat{k}$$

Collinear Vector



$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{z_1}{z_2}$$

Equal Vector

$$|\vec{a}| = \frac{2}{\frac{1}{2}} \text{ Unit}$$

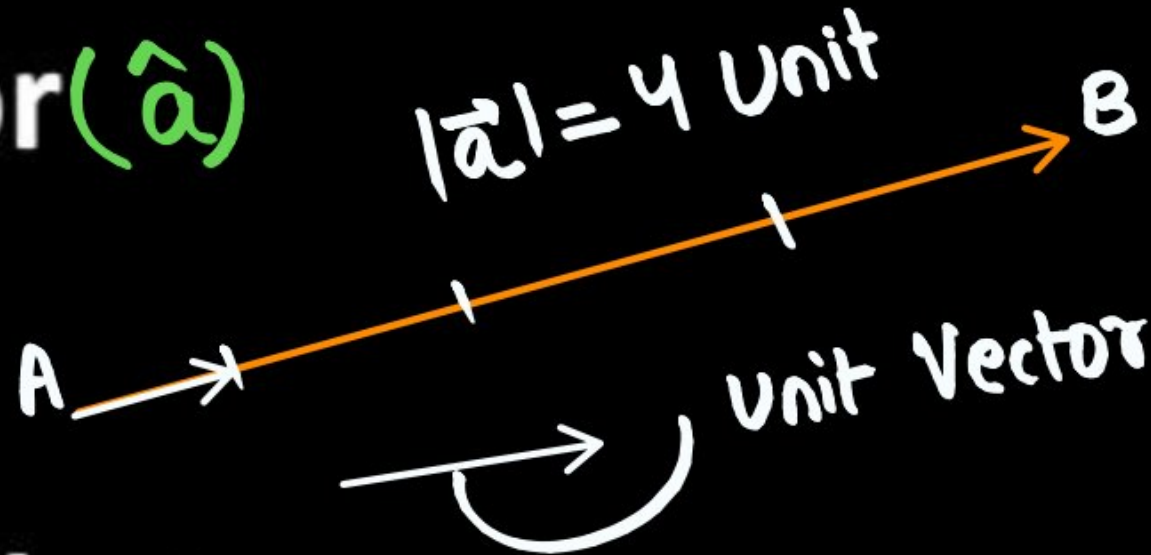
$$|\vec{b}| = 2 \text{ Unit}$$

$$\vec{a} = \vec{b}$$

Magnitude & Direction. Same equal vector

Unit Vector ($\hat{a}$)

$$|\hat{a}| = 1$$

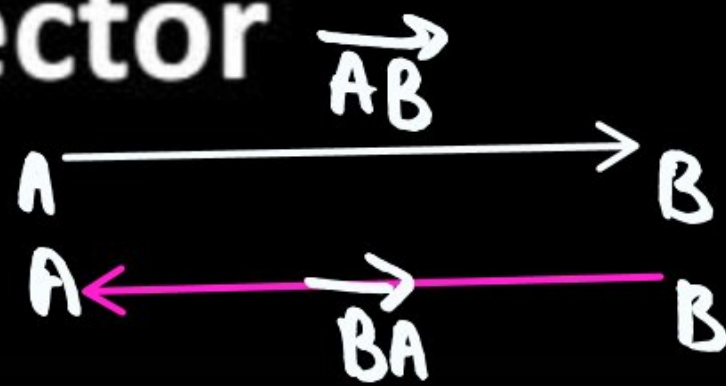


$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

Negative Vector



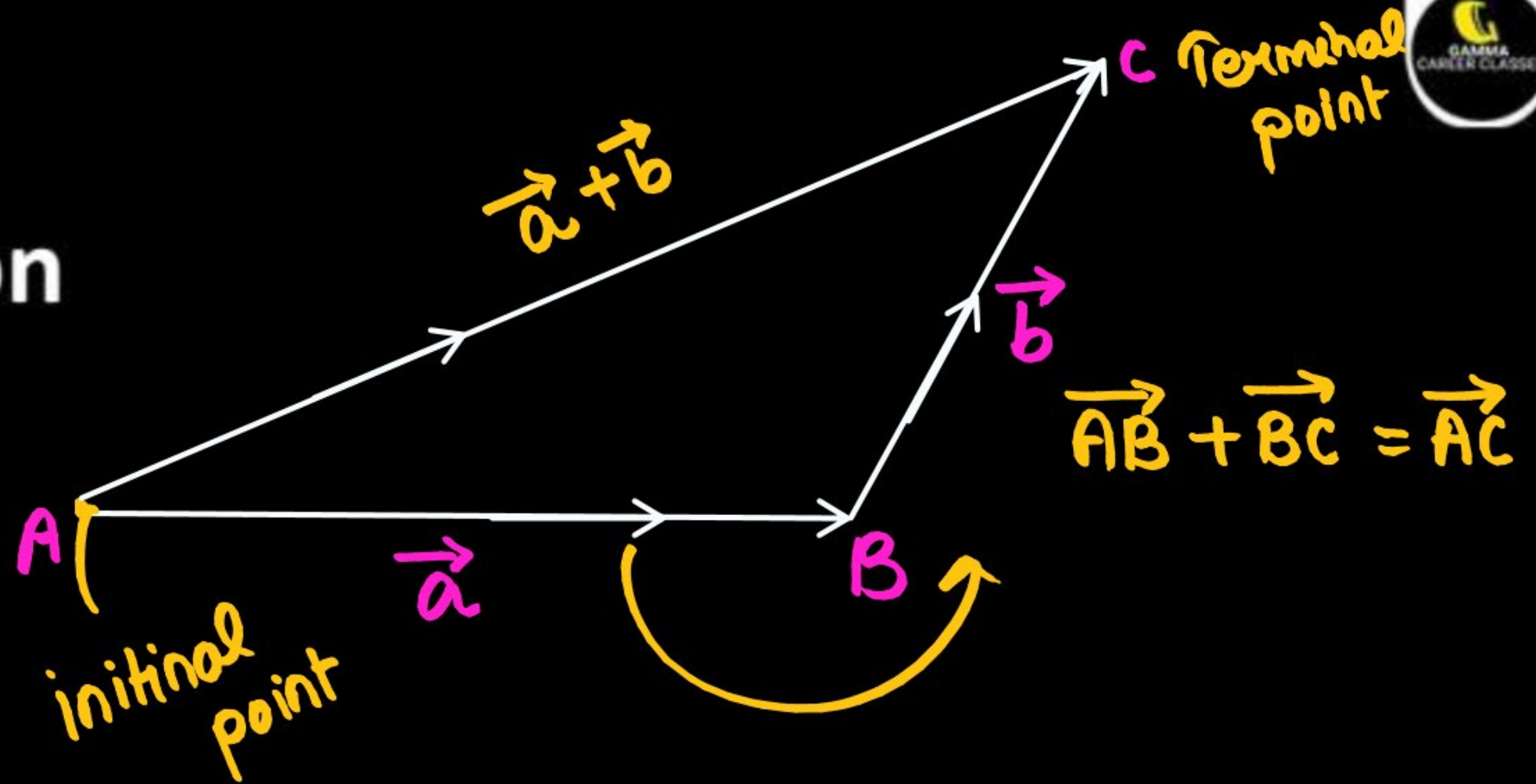
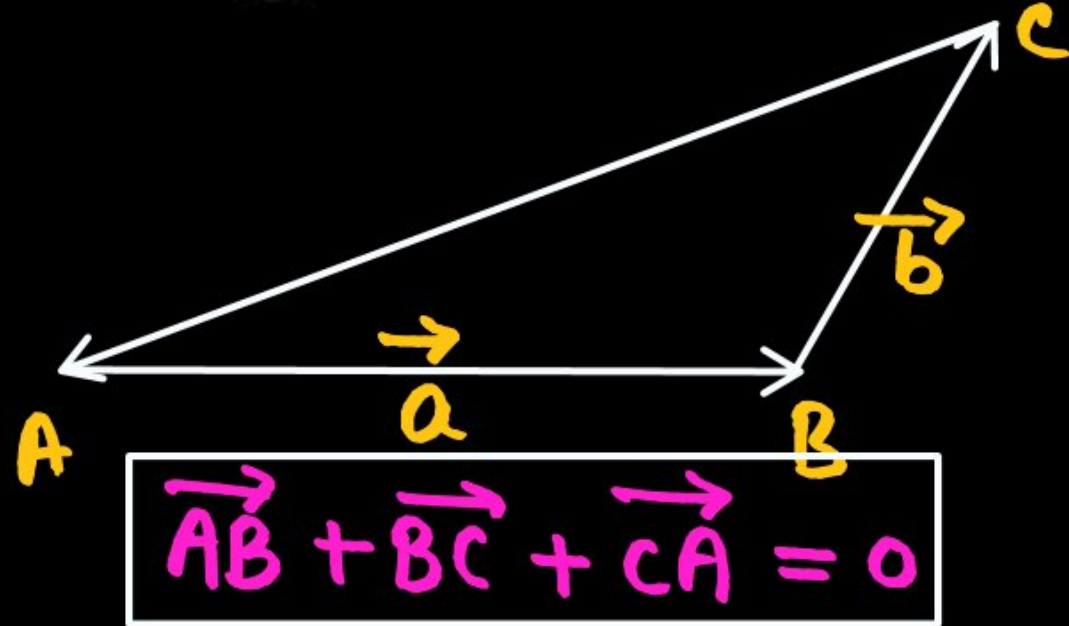
$$\vec{AB} = -\vec{BA}$$

$$\vec{AB} + \vec{BA} = 0$$

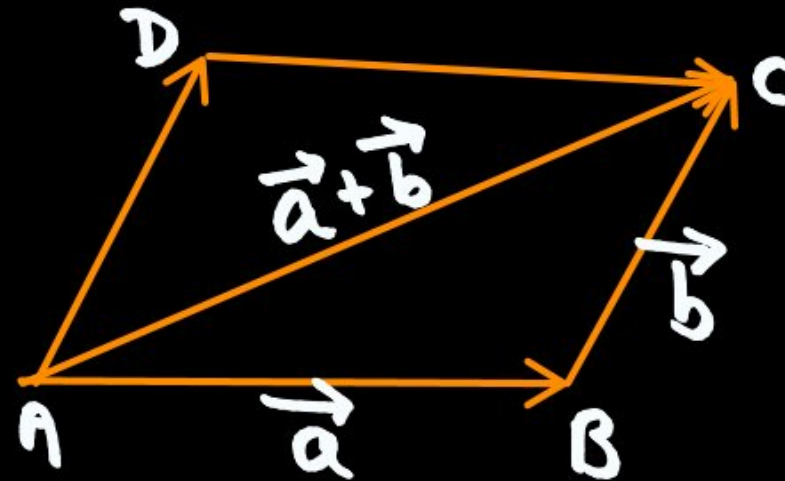
$$\hat{a} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

Addition of Vector

Triangle law of addition



Parallelogram law of vector addition



Multiplication of a Vector by Scalar



$$|\vec{a}| = 2 \text{ Unit}$$

$$\boxed{2\vec{a}}$$

Scalar

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Scalar

$$3\vec{a} = 3[2\hat{i} + 3\hat{j} + 4\hat{k}]$$

$$\boxed{3\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}}$$

Components of Vector



$\hat{i}$ → Unit vector along to x-axis

$\hat{j}$ → Unit vector along to y-axis

$\hat{k}$ → Unit vector along to z-axis

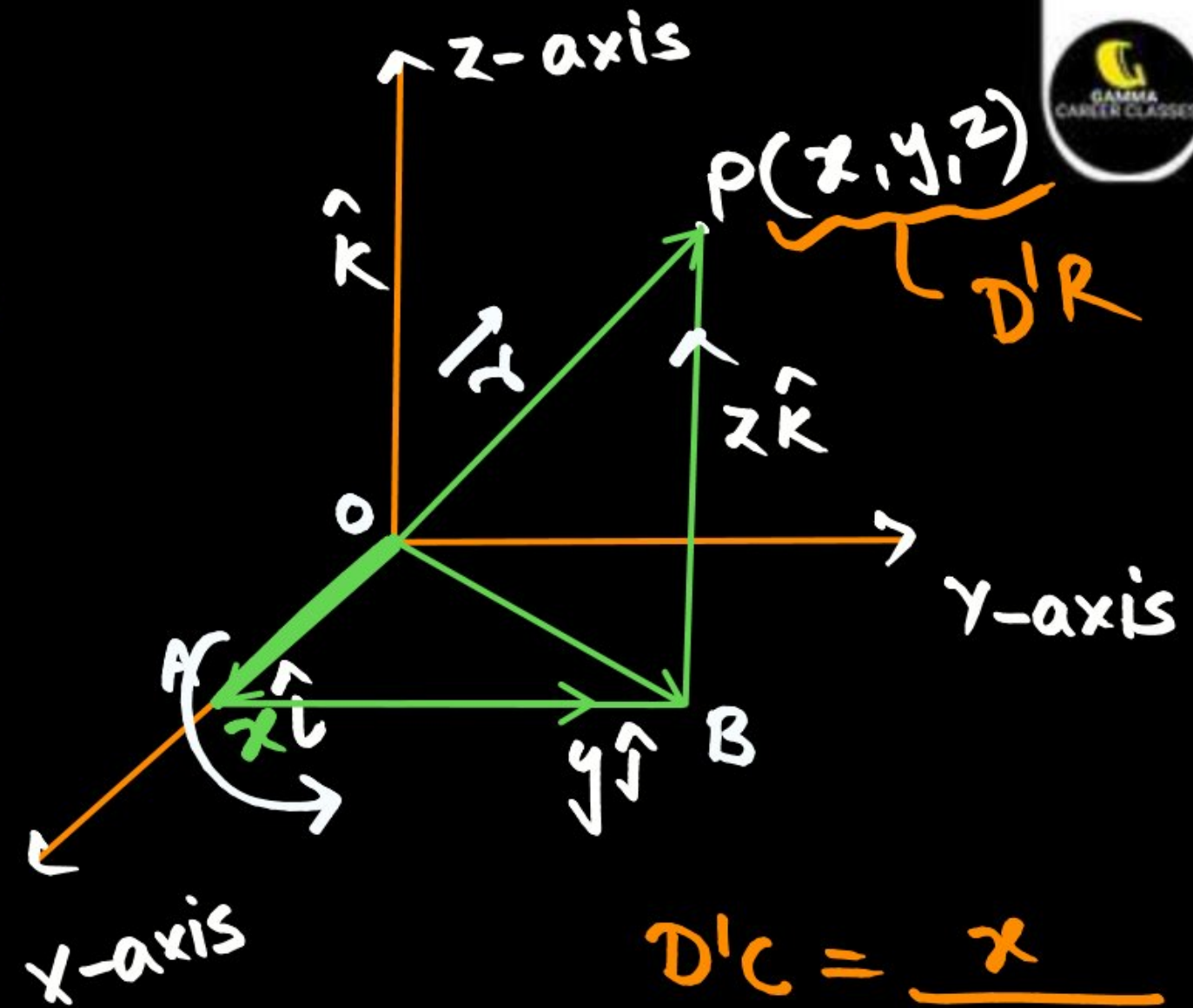
$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{OB} = x\hat{i} + y\hat{j}$$

$$\vec{OP} = \vec{OB} + \vec{BP}$$

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



$$D'C = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$= \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$
$$= \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$(3, 2, 1)$ — Scalar components

$(3\hat{i}, 2\hat{j}, \hat{k})$ — Vector components.

Show that the vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

$$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k} \quad a_1 = 2 \quad b_1 = -3 \quad c_1 = 4$$

$$\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} \quad a_2 = -4 \quad b_2 = 6 \quad c_2 = -8$$

$$\frac{2}{-4} = \frac{-3}{6} = \frac{4}{-8}$$

$$-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$$

Find λ , if the vector $3\hat{i} + \hat{j} - 5\hat{k}$ and $3\hat{i} + \hat{j} - \lambda\hat{k}$ are parallel

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\vec{a} = 3\hat{i} + \hat{j} - 5\hat{k}$$

$$\vec{b} = 3\hat{i} + \hat{j} - \lambda\hat{k}$$

$$\frac{3}{3} = \frac{1}{1} = \frac{-5}{-\lambda}$$

$$1 = \frac{5}{\lambda}$$

$$\lambda = 5$$

Find the Unit Vector

$$\hat{a}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$$

$$|\vec{a}| = \sqrt{3^2 + 2^2 + 1^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$|\vec{a}| = \sqrt{14}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\hat{a} = \frac{3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}}$$

Find the Unit Vector $A(3,1,2)$

$$\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{(3)^2 + (1)^2 + (2)^2}$$

$$= \sqrt{9 + 1 + 4}$$

$$|\vec{a}| = \sqrt{14}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{3\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{14}}$$

$$\hat{a} = \frac{3}{\sqrt{14}}\hat{i} + \frac{1}{\sqrt{14}}\hat{j} + \frac{2}{\sqrt{14}}\hat{k}$$

$$\text{D'R } \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}$$

$$\text{D'C} = \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}$$

D'R and D'C
proportional

Find Unit Vector in the direction of vector $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$



$$\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\begin{aligned} |\vec{a}| &= \sqrt{(3)^2 + (2)^2 + (1)^2} \\ &= \sqrt{9 + 4 + 1} \\ &= \sqrt{14} \end{aligned}$$

$$\hat{a} = \frac{3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}}$$

Write a vector of magnitude 18 units in the direction of vector $\hat{i} - 2\hat{j} - 2\hat{k}$



Important

let

$$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$$|\vec{a}| = 18 \text{ unit}$$

2 unit
Magnitude.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{1+4+4}} = \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{9}} = \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{3}$$

$$\vec{a} = \underbrace{|\vec{a}|}_{\text{Magnitude}} \underbrace{\hat{a}}_{\text{Direction}}$$

$$\vec{a} = 18 \left[\frac{\hat{i} - 2\hat{j} - 2\hat{k}}{3} \right] = 6\hat{i} - 12\hat{j} - 12\hat{k}$$

Find the unit vector in the direction of the sum of vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \quad \vec{b} = 4\hat{i} - 3\hat{j} + 2\hat{k}$$

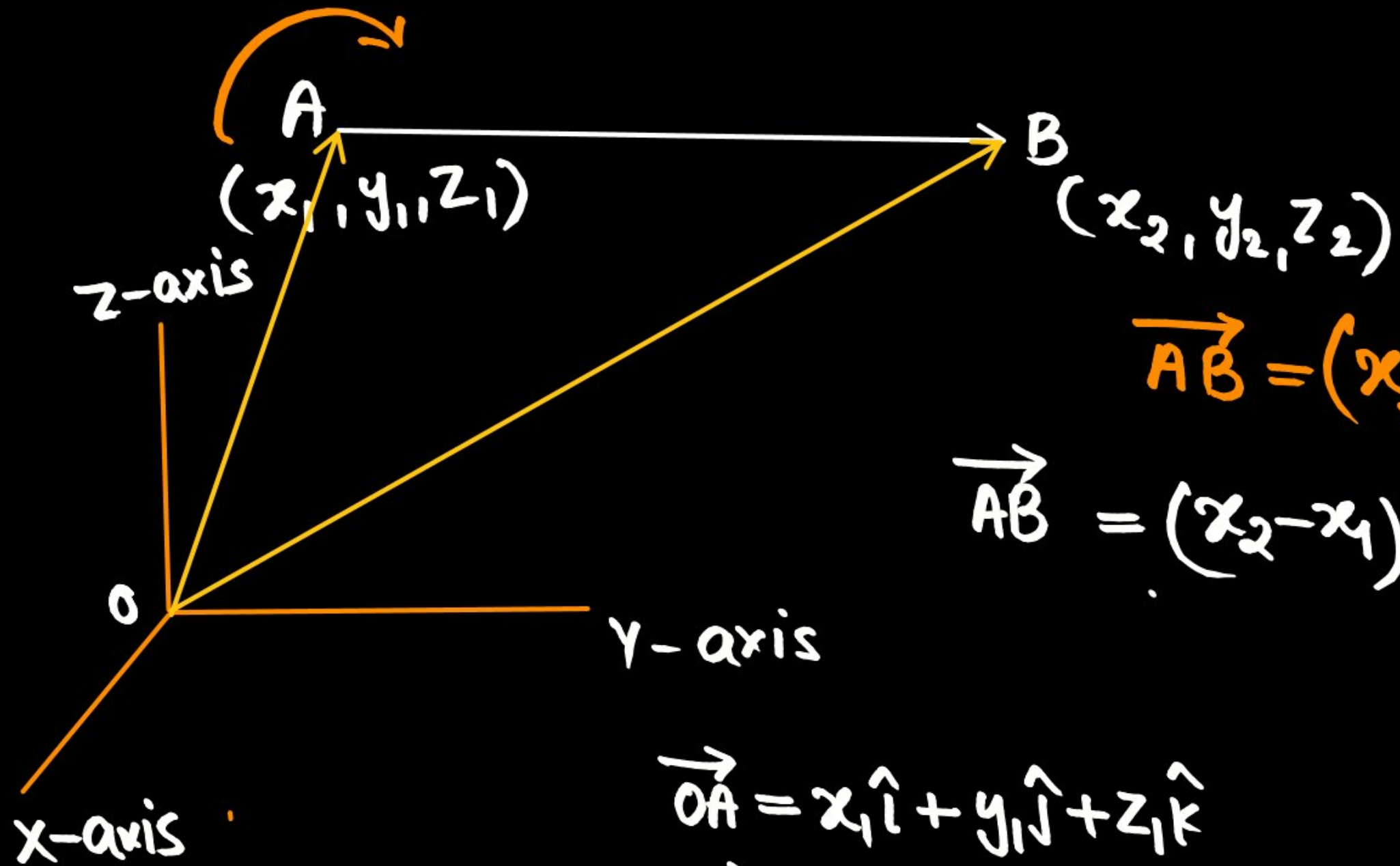
$$\vec{a} + \vec{b} = 2\hat{i} + 4\hat{i} + 3\hat{j} - 3\hat{j} - \hat{k} + 2\hat{k}$$

$$\vec{a} + \vec{b} = 6\hat{i} + 0\hat{j} + \hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{(6)^2 + 0 + (1)^2} = \sqrt{37}$$

$$\text{Unit vector of } \vec{a} + \vec{b} = \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{6\hat{i} + \hat{k}}{\sqrt{37}} \quad \text{Ans}$$

Vector Joining Two Point



$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\vec{OA} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{OB} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

Find the vector joining the points P(3,3,0) and Q(-1,2,-3) directed from

$$\vec{OP} = 3\hat{i} + 3\hat{j} + 0\hat{k}$$

$$\vec{OQ} = -\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (-1-3)\hat{i} + (2-3)\hat{j} + (-3-0)\hat{k}$$

$$\boxed{\vec{PQ} = -4\hat{i} - \hat{j} - 3\hat{k}}$$

Find the terminal point of the vector $\overrightarrow{PQ}$ whose initial point is $P(-2,5,0)$ and components along coordinate axes are 2, -3 and 7 respectively.

Important

$$\overrightarrow{PQ} = 2\hat{i} - 3\hat{j} + 7\hat{k}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$2\hat{i} - 3\hat{j} + 7\hat{k} = x\hat{i} + y\hat{j} + z\hat{k} - (-2\hat{i} + 5\hat{j} + 0\hat{k})$$

$$2\hat{i} - 3\hat{j} + 7\hat{k} = (x+2)\hat{i} + (y-5)\hat{j} + (z-0)\hat{k}$$

$$x+2=2$$

$$x=0$$

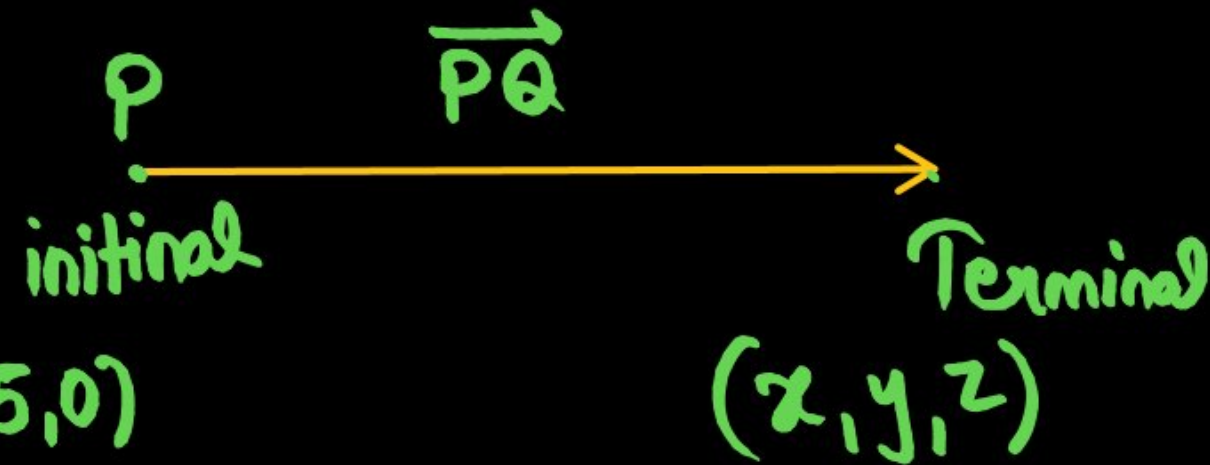
$$y-5=-3$$

$$y=-3+5=2$$

$$z-0=7$$

$$z=7$$

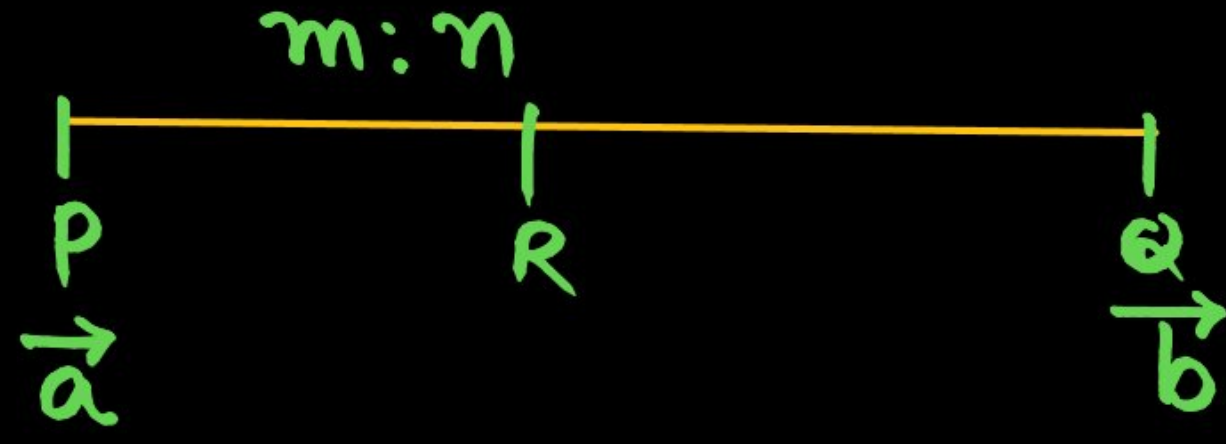
$$(0, 2, 7)$$



$$\overrightarrow{OP} = -2\hat{i} + 5\hat{j} + 0\hat{k}$$

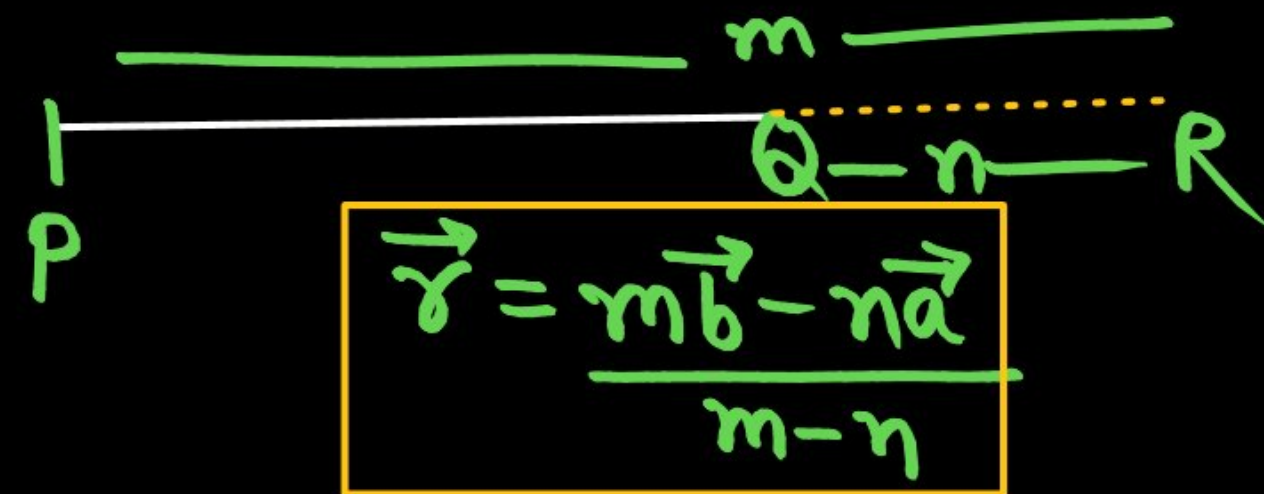
$$\overrightarrow{OQ} = x\hat{i} + y\hat{j} + z\hat{k}$$

Section Formula



internal point

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$



Externally point

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2:1

i) Internally

Ratio = 2:1 $m:n$

$$\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{b} = -\hat{i} + \hat{j} + \hat{k}$$

$$\vec{r} = \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3}$$

ii) Externally

$$\frac{-2\hat{i} + 2\hat{j} + 2\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{3}$$

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

(H.W)

(i)

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

$$= \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2 + 1}$$

Find the mid-point of the vector joining the point P and Q having position vectors $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $4\hat{i} + \hat{j} - 2\hat{k}$ respectively

$$\vec{OP} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\vec{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$$

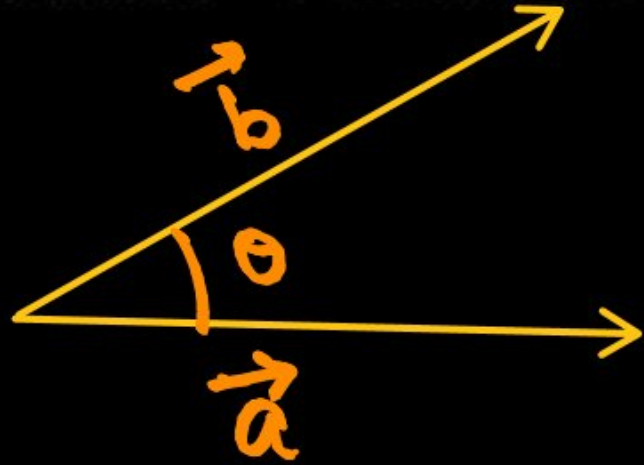


$$\vec{r} = \frac{4\hat{i} + \hat{j} - 2\hat{k} + 2\hat{i} + 3\hat{j} - 4\hat{k}}{2}$$

$$\vec{r} = \frac{\vec{b} + \vec{a}}{2}$$

$$\vec{r} = \frac{6\hat{i} + 4\hat{j} - 6\hat{k}}{2} = 3\hat{i} + 2\hat{j} - 3\hat{k}$$

Product of two vectors



$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

Important result

(i)

$$\theta = 90^\circ$$

$$\vec{a} \cdot \vec{b} = 0$$

(ii)



$$\theta = 0^\circ$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$$

$$\begin{aligned}\vec{a} \cdot \vec{a} &= |\vec{a}| |\vec{a}| = |\vec{a}|^2 \\ \vec{b} \cdot \vec{b} &= |\vec{b}| |\vec{b}| = |\vec{b}|^2\end{aligned}$$

If $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 7\hat{k}$ find the $\vec{a} \cdot \vec{b}$ and $\vec{b} \cdot \vec{a}$

$$\vec{a} \cdot \vec{b} = (2\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 7\hat{k})$$

$$= 6 - 12 - 35$$

$$\vec{a} \cdot \vec{b} = -41$$

$$\vec{b} \cdot \vec{a} = (3\hat{i} + 4\hat{j} - 7\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 5\hat{k})$$

$$= (6 - 12 - 35)$$

$$\vec{b} \cdot \vec{a} = -41$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\hat{i} \cdot \hat{i} = 1$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = 0$$

$$\hat{j} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{i} = 0$$

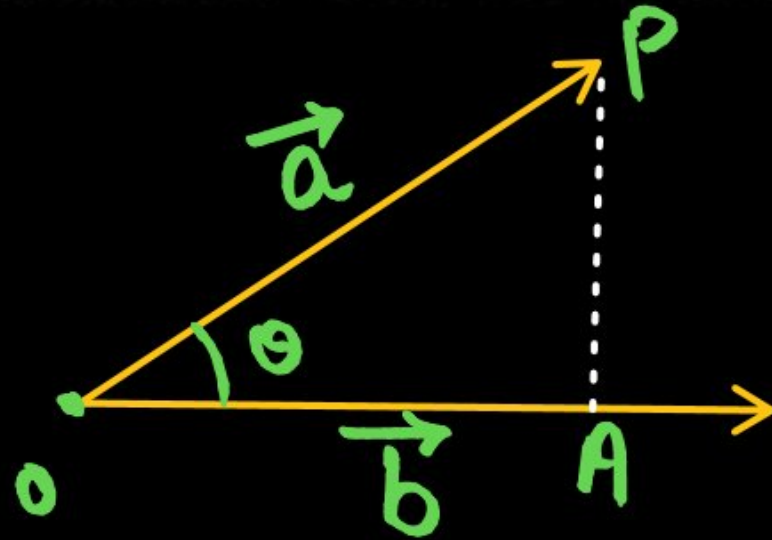
Find the angle between vectors $\vec{a}$ and $\vec{b}$ such that
 $|\vec{a}| = |\vec{b}| = 3$ $\vec{a} \cdot \vec{b} = 1$

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{1}{3 \times 3}\end{aligned}$$

$$\cos \theta = \frac{1}{9}$$

$$\theta = \cos^{-1} \frac{1}{9}$$

Projection of a vector on a line.



$$\begin{aligned} \text{Projection of } \vec{a} \text{ on } \vec{b} \\ &= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \end{aligned}$$

length of Shadow of given vector

$$\frac{|\vec{OP}|}{|\vec{a}|} = \cos \theta$$

$$|\vec{OP}| = |\vec{a}| \cos \theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = |\vec{a}| \cos \theta$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = |\vec{OP}|$$

Find the projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Important

$$\text{Projection of vector } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{(\hat{i} + 3\hat{j} + 7\hat{k})(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{(2)^2 + (-3)^2 + (6)^2}}$$

$$= \frac{2 - 9 + 42}{\sqrt{4 + 9 + 36}} = \frac{35}{\sqrt{49}} = \frac{35}{7} = 5$$

$$\begin{aligned} |\vec{a} + \vec{b}|^2 &= (\vec{a} + \vec{b})(\vec{a} + \vec{b}) \\ &= \vec{a}(\vec{a} + \vec{b}) + \vec{b}(\vec{a} + \vec{b}) \\ &= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} \end{aligned}$$

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$$

If $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a} \cdot \vec{b} = 4$. Find $|2\vec{a} - 3\vec{b}|$

$$|2\vec{a} - 3\vec{b}|^2 = |2\vec{a}|^2 + |3\vec{b}|^2 - 2 \cdot 2\vec{a} \cdot 3\vec{b}$$

$$= 4|\vec{a}|^2 + 9|\vec{b}|^2 - 12\vec{a} \cdot \vec{b}$$

$$= 4 \times (2)^2 + 9 \times (3)^2 - 12 \times 4$$

$$= 4 \times 4 + 9 \times 9 - 48$$

$$= 16 + 81 - 48$$

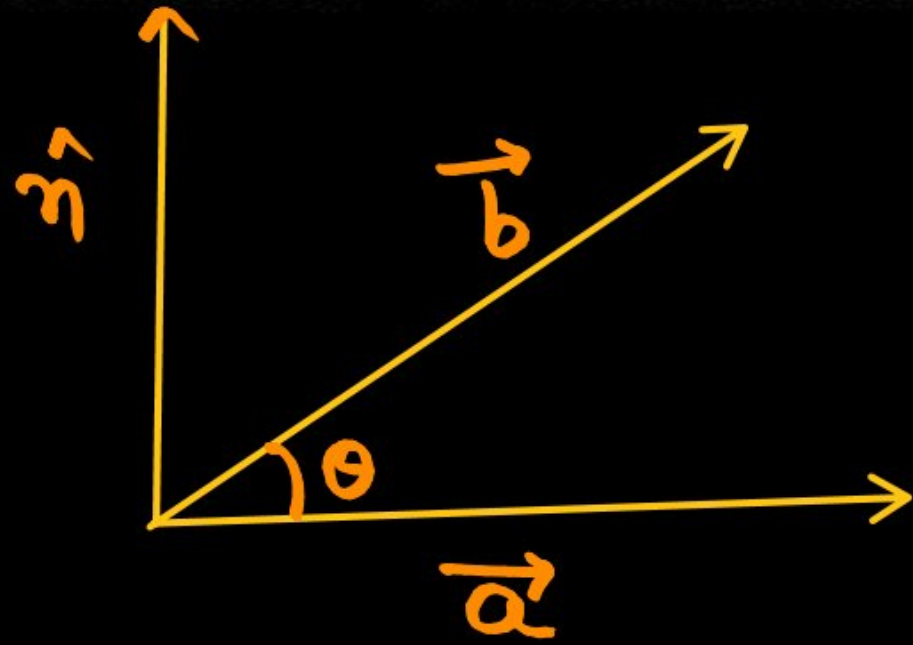
$$= 97 - 48$$

$$|2\vec{a} - 3\vec{b}|^2 = 49$$

$$|2\vec{a} - 3\vec{b}| = \sqrt{49}$$

$$|2\vec{a} - 3\vec{b}| = 7$$

Vector Product of two Vector



$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times \vec{b} = \underbrace{|\vec{a}| |\vec{b}| \sin \theta}_{\text{Magnitude}} \underbrace{\hat{n}}_{\text{Direction}}$$

Magnitude

Direction.

Important point

(i) $\theta = 90^\circ$

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \hat{n}$$

(ii) $\theta = 0^\circ$

$$\vec{a} \times \vec{b} = 0$$

(iii) $\hat{i} \times \hat{i} = 0$ $\hat{k} \times \hat{k} = 0$
 $\hat{j} \times \hat{j} = 0$

$$\hat{i} \times \hat{j} = \hat{k}$$

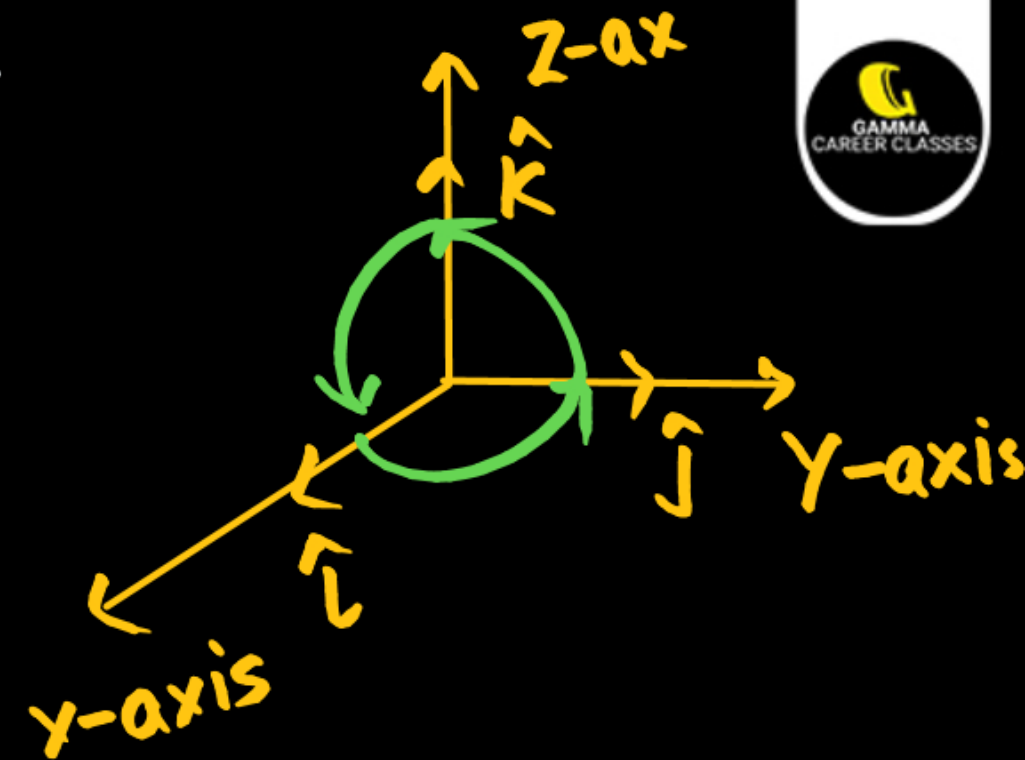
$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$



$$\vec{a} = a_1 \hat{i} + b_1 \hat{j} + c_1 \hat{k}$$

$$\vec{b} = a_2 \hat{i} + b_2 \hat{j} + c_2 \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$, $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$ then find the a unit vector perpendicular to both of the vector $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

$$\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j}) = -\hat{i} + \hat{j} + \hat{k}$$

$$\vec{c} - \vec{b} = (3\hat{i} - 4\hat{j} - 5\hat{k}) - (2\hat{i} + \hat{j}) = \hat{i} - 5\hat{j} - 5\hat{k}$$

$$(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix}$$

$$(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})$$

$$= \hat{i}(-5+5) - \hat{j}(5-1) + \hat{k}(5-1)$$

$$= 0\hat{i} - 4\hat{j} + 4\hat{k}$$

$$|(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})| =$$

$$= \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+16}$$

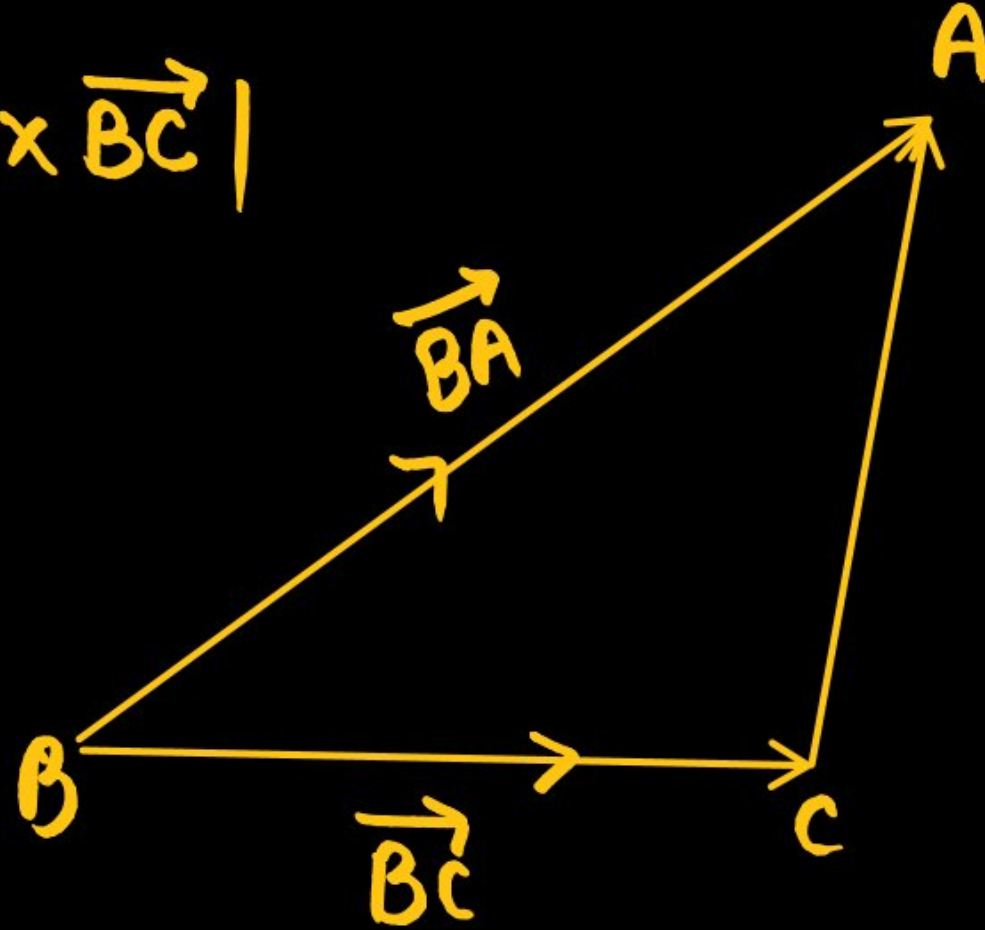
$$= \sqrt{32}$$

$$(\vec{a}-\vec{b}) \times (\vec{c}-\vec{b}) = \frac{-4\hat{j}+4\hat{k}}{\sqrt{32}}$$

$$(\vec{a}-\vec{b}) \times (\vec{c}-\vec{b}) = \frac{-4\hat{j}+4\hat{k}}{4\sqrt{2}}$$

Area of triangle by Vector

$$\text{Area of } \Delta = \frac{1}{2} |\vec{BA} \times \vec{BC}|$$



Using vectors find the area of the triangle whose Vertices are $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$

$$\vec{OA} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{OB} = \hat{i} - \hat{j} - 3\hat{k}$$

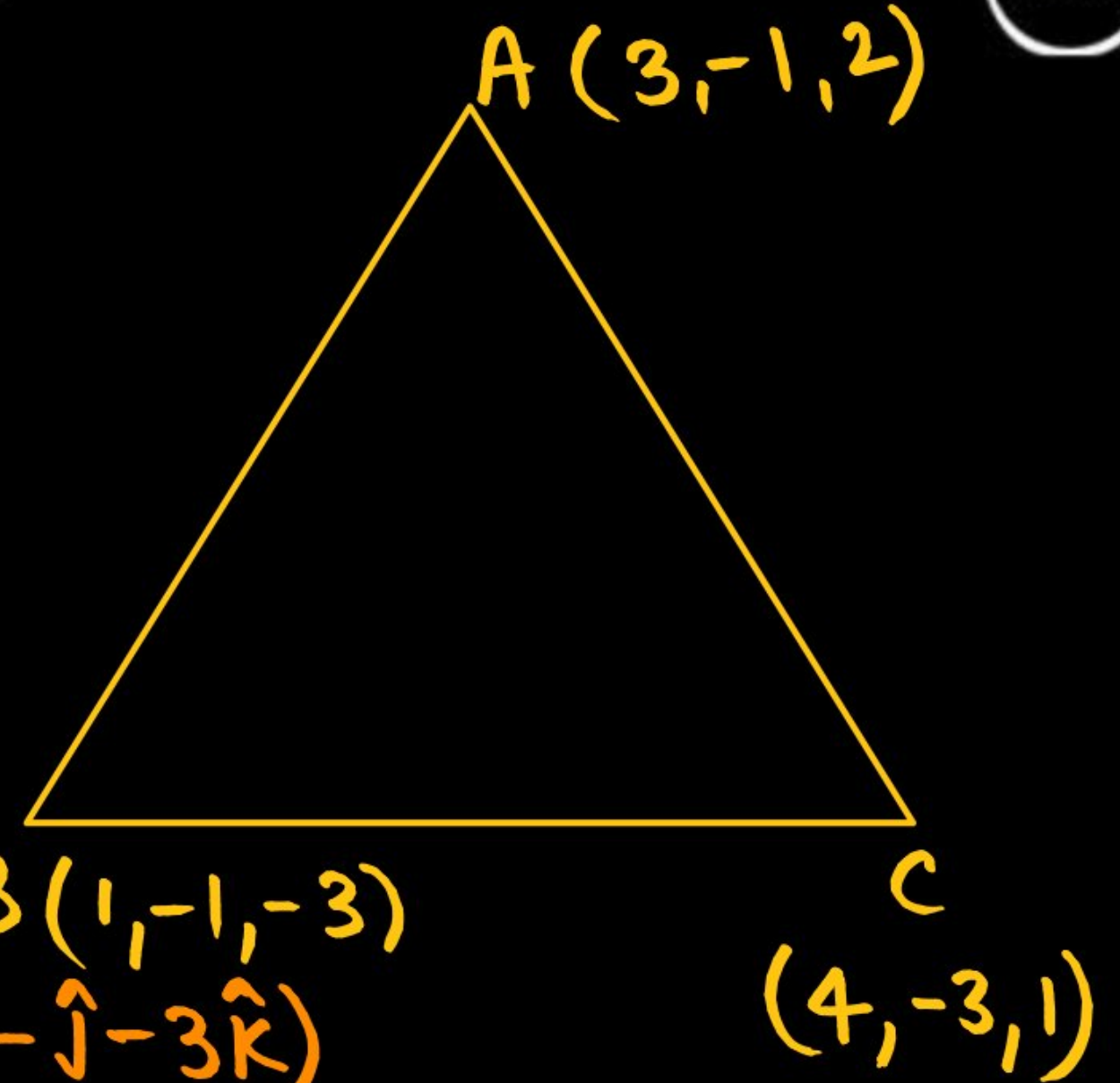
$$\vec{OC} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\text{Area of } \Delta = \frac{1}{2} |\vec{BA} \times \vec{BC}|$$

$$\vec{BA} = \vec{OA} - \vec{OB} = 3\hat{i} - \hat{j} + 2\hat{k} - (\hat{i} - \hat{j} - 3\hat{k})$$

$$\vec{BA} = 2\hat{i} - 0\hat{j} + 5\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = 4\hat{i} - 3\hat{j} + \hat{k} - (\hat{i} - \hat{j} - 3\hat{k}) = 3\hat{i} - 2\hat{j} + 4\hat{k}$$



$$\vec{BA} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

$$\hat{i}(0+10) - \hat{j}(8-15) + \hat{k}(-4-0)$$

$$\vec{BA} \times \vec{BC} = 10\hat{i} + 7\hat{j} - 4\hat{k}$$

$$|\vec{BA} \times \vec{BC}| = \sqrt{10^2 + 7^2 + (-4)^2}$$

$$= \sqrt{100 + 49 + 16}$$

$$= \sqrt{165}$$

$$\text{Area of } \Delta = \frac{1}{2} |\vec{BA} \times \vec{BC}|$$

$$\text{Area} = \frac{1}{2} \times \sqrt{165} \text{ Sq. Unit}$$